

# Thermofield qubits, generalized expectations and quantum information protocols

T. Prudêncio

*Coordination in Science and Technology - CCCT,  
Federal University of Maranhão - UFMA,  
Campus Bacanga, 65080-805, São Luis-MA, Brazil.*

T. M. Rocha Filho

*International Centre for Condensed Matter Physics - ICCMP & Institute of Physics,  
University of Brasília - UnB,  
CP: 04455, 70919-970, Brasília - DF, Brazil.*

A. E. Santana

*International Centre for Condensed Matter Physics - ICCMP & Institute of Physics,  
University of Brasília - UnB,  
CP: 04455, 70919-970, Brasília - DF, Brazil.*

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## Abstract

Thermofield dynamics (TFD) approach is a real time quantum field method for dealing with finite temperature quantum states in a purified version of usual density operator formalism at finite temperature. In the domain of quantum information, TFD represents a quite promising direction for dealing with qubits under thermal influence and can also be associated to Gaussian states. Here, we propose a generalized TFD mean expectation for the case of thermofield qubits considering the action of gate operators. We propose quantum teleportation protocols involving thermofield states, considering thermal-to-thermal and thermal-to-non-thermal transferring cases. In particular, we discuss the case in which Alice and Bob are at different temperatures. Action of gate operators on the result of the Mandel parameter for thermofields and on Gibbs-like density operators are also discussed. The no-cloning and non-broadcasting theorems in TFD are also considered and cases

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*Email address:* prudencio.thiago@ufma.br (T. Prudêncio)

of superposed thermofield states and maps connecting thermofield vacua at different temperatures are also addressed and associated to metastable and non-equilibrium scenarios.

*Keywords:* Themofields, quantum states, qubits.

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## 1. Introduction

Generation of thermal states by doubling the degrees of freedom in a Hilbert space accompanied by the action of a temperature dependent Bogoliubov transformation, thermofield dynamics (TFD)<sup>1,2,3</sup>, is a natural approach for dealing with finite temperature quantum states in a purified version of usual density operator formalism in a finite temperature scenario. Proposed by Takahashi and Umezawa<sup>4</sup> as a real time quantum field theory at finite temperature, TFD has been applied in different contexts<sup>5,6,7,8,9,10,11,12,13</sup>, ranging high energy physics<sup>14,15,16,17,18,19,20,21,22</sup>, quantum statistical mechanics<sup>23,24</sup>, quantum optics<sup>25,26,27,28,29,30,31</sup> and condensed matter<sup>32,33,34</sup>. Particularly, the thermofield state corresponding to the vacuum at finite temperature has an expectation value equivalent to the equilibrium quantum statistical measure. This state is associated to a thermal density operator at a given temperature,  $T = \beta^{-1}$ . The physical correspondence of such thermofield vacuum with a mixed state associated to a given density operator was established previously and the physical meaning of the doubling has been fully identified (see<sup>36</sup> for a recent review). Considering an arbitrary mixed state, the non-tilde creation operators are identified with addition photon states, while tilde creation operators are associated with subtraction photon states<sup>35</sup>. On the other hand, realization of quantum information processing requires implementation of gate operations, incorporating transmission and state manipulation in a complete quantum computational scheme. One important role is represented by the quantum teleportation (QT), that dates back to Aharonov and Albert's result, in which nonlocality in a quantum system can be measured without violating causality<sup>38</sup>, the no-cloning theorem<sup>39</sup> and the famous propose by Bennett et al.<sup>40</sup> describing a protocol for the transmission over spatial distances with reconstruction of a quantum state, followed by an avalanche of other important theoretical proposes and experimental realizations (see for instance<sup>41,42,43,44,45,46,47,48,49,50,51,52,53,54,55,56,57,58,59,60,62,61,63</sup>). In particular, the route for large scale quantum communication has been started with photonic and ionic single qubits, polarized states of at least two-photons<sup>64,65</sup>, in-

creasing the range of fidelities above 0.8, covering distances above  $\sim 140\text{km}$ <sup>66</sup> and minimizing loss effects during transmission<sup>67,68,69,70,71</sup>. A fundamental link to quantum computation was established by Gottesman and Chuang proving that QT can be used as a universal primitive, reducing resource requirements for quantum computers and unifying known protocols for fault-tolerant quantum computation<sup>72</sup>.

Bit-encodings using thermal logical bits, with proposes in thermal logical gates and thermal transistors<sup>73</sup>, reinforce the fact that the thermal properties can be used as a resource for transmission of information and computation.

Quantum circuits incorporating incoherent resources<sup>74</sup>, fault-tolerant logical gates<sup>75</sup>, systems tolerant to decoherence arising from local noise<sup>76</sup> and controllability of qubits<sup>77</sup> are examples of efforts to circumvent the inevitable presence of noise and environment effects in real quantum computation scenarios, justifying the search for new approaches for quantum information protocols. These routes can be extended in particular in the framework of quantum states at finite temperature, where TFD and other finite temperature methods can enter into play.

Indeed, TFD approach to quantum information represents a quite promising direction for dealing with qubits under a thermal environment influence. In quantum information protocols, in particular QT, TFD has the possibility of dealing with non-locality and entanglement at finite temperature scenario making use of the algebraic structure of such thermofield states. This route for the investigation of non-locality in thermal environments brings new features relating quantum information protocols in a thermofield scenario and have been explored in some recent proposes associated to maximally entangled states<sup>78,79,80</sup>, no-cloning theorem<sup>81</sup>, quantum gates<sup>82</sup>. Thermofield states are also used for description of eternal anti-de Sitter (AdS) black holes<sup>83,84</sup> with association to quantum complexity<sup>85</sup> and can also be associated to Gaussian states<sup>86</sup>.

In this work, the effect of temperature is implemented via TFD, by means of which we reconsider the expectation relations of thermal states for the case of thermofield qubits, deriving a generalized relation for this case under the action of gate operations. We also propose protocols where thermal-to-thermal and thermal-to-non-thermal quantum transfers and QT are realized with thermofield states. In particular, the case in which Alice and Bob are at different temperatures is also considered and discussed. We also discuss the TFD formulations involving the Mandel parameter and Gibbs like operators under the action of gate operations. No-cloning theorem for thermofields<sup>81</sup>

and the problem of non-broadcasting<sup>88</sup> in temperature dependent situations are investigated, where the connection among different thermofield states at different temperatures and how such a method can be used in the domain of metastable and non-equilibrium states are also discussed.

The paper is organized as follows: In section II, we discuss generalized thermofield mean expectations and the action of gate operations on a thermofield qubits. In section III, we propose QT of thermofield qubits. In section IV, we consider changing the Mandel parameter of a thermofield state under gate operations. In Section V, we consider Gibbs-like density operators under gate operations. In section VI, we discuss the no-cloning theorem for TFD. In section VII, we consider maps connecting thermofield vacua, no-broadcasting theorem and superposition of thermofield vacua. We also consider superpositions of thermofield states at different temperatures and discuss their application in metastable and non-equilibrium scenarios. Finally, in section VIII, we address our concluding remarks.

## 2. Generalized thermofield mean expectations and the action of gate operations on a thermofield qubit

We start with a superposition of thermofield states

$$|\psi(\beta)\rangle = \sum_{j \in Z_{n+1}} a_j |j(\beta)\rangle, \quad (1)$$

where  $a_j$  are arbitrary complex numbers satisfying  $\sum_{j \in Z_{n+1}} |a_j|^2 = 1$  and the set  $Z_{n+1} = \{0, 1, \dots, n\}$  is the set of positive integer numbers mod  $n+1$ . In the case of  $Z_2$ , the set obeys simple algebra with  $1+1 = 0+0 = 0$ ,  $1+0 = 0+1 = 1$ . The states  $|j(\beta)\rangle$  are  $j$ -order excitations from the thermofield vacuum  $|0(\beta)\rangle$ , by the action of a thermal creation operator<sup>35</sup>. The association of the expectation value of thermal vacuum with the statistical mean is given by the following relation

$$\langle \hat{\mathcal{O}} \rangle = \langle 0(\beta) | \hat{\mathcal{O}} | 0(\beta) \rangle = Tr(\hat{\rho}^{th} \hat{\mathcal{O}}), \quad (2)$$

where  $\hat{\mathcal{O}}$  is an operator acting on the non-tilde sector of the thermofield and  $\hat{\rho}^{th}$  is the associated thermal density operator. In the case of the bosonic state with associated number state  $\hat{\mathcal{O}} = \hat{n}$ , the thermofield vacuum implies

$$\bar{n} = \langle \hat{n} \rangle = \langle 0(\beta) | \hat{a}^\dagger \hat{a} | 0(\beta) \rangle = Tr(\hat{\rho}^{th} \hat{n}), \quad (3)$$

where  $\hat{\rho}^{th}$  can be decomposed in terms of number states

$$\hat{\rho}^{th} = \sum_{n=0}^{\infty} \frac{\bar{n}^n}{(\bar{n} + 1)^{n+1}} |n\rangle\langle n|. \quad (4)$$

For the case of associated modes with energy frequency  $\omega$ , the following Bose-Einstein distribution

$$\bar{n} = \frac{1}{1 + e^{\beta\omega}} \quad (5)$$

is associated to the thermal state and maximizes the von Neumann entropy  $S(\hat{\rho}) = -Tr(\hat{\rho} \log \hat{\rho})$ ,

$$S(\hat{\rho}^{th}) = \max S(\hat{\rho}). \quad (6)$$

Considering the state in Eq. (1), the expectation value for the observable  $\hat{\mathcal{O}}$  can be defined by means of relation

$$\langle \psi(\beta) | \hat{\mathcal{O}} | \psi(\beta) \rangle = \sum_{j,j' \in Z_n} a_j a_{j'}^* \langle j(\beta) | \hat{\mathcal{O}} | j'(\beta) \rangle, \quad (7)$$

where  $a_{j'}^*$  is the complex conjugated of  $a_{j'}$ .

By restricting ourselves to the set  $Z_2$ , forming a thermofield qubit of the thermal vacuum and its first thermal excitation, the Bogoliubov relation  $\hat{c}^\dagger = u(\beta)\hat{\mathbf{c}}^\dagger(\beta) + v(\beta)\tilde{\mathbf{c}}(\beta)$ , where  $\hat{\mathbf{c}}(\beta)$  and  $\tilde{\mathbf{c}}(\beta)$  are the corresponding non-tilde and tilde thermofield operators of annihilation, can be used to write the association between the excited non-tilde thermofield and the thermofield vacuum by means of the following relation

$$|1(\beta)\rangle = \frac{\hat{c}^\dagger}{u(\beta)} |0(\beta)\rangle. \quad (8)$$

Using this expression, we rewrite Eq. (7),  $n = 2$ , explicitly in terms of traces

$$\begin{aligned}
\langle \hat{\mathcal{O}} \rangle_{\psi(\beta)} &= \sum_{j,j' \in Z_2} a_j a_{j'}^* \langle 0(\beta) | (\frac{\hat{c}}{u(\beta)})^j \hat{\mathcal{O}} (\frac{\hat{c}^\dagger}{u(\beta)})^{j'} | 0(\beta) \rangle, \\
&= \sum_{j,j' \in Z_2} \frac{a_j}{u(\beta)^j} \frac{a_{j'}^*}{u(\beta)^{j'}} \langle 0(\beta) | \hat{c}^j \hat{\mathcal{O}} \hat{c}^{\dagger j'} | 0(\beta) \rangle, \\
&= \sum_{j,j' \in Z_2} \frac{a_j}{u(\beta)^j} \frac{a_{j'}^*}{u(\beta)^{j'}} \text{Tr}(\hat{\rho} \hat{c}^j \hat{\mathcal{O}} \hat{c}^{\dagger j'}), \\
&= \sum_{j,j' \in Z_2} \frac{a_j}{u(\beta)^j} \frac{a_{j'}^*}{u(\beta)^{j'}} \text{Tr}(\hat{c}^{\dagger j'} \hat{\rho} \hat{c}^j \hat{\mathcal{O}}), \tag{9}
\end{aligned}$$

where  $\langle \hat{\mathcal{O}} \rangle_{\psi(\beta)} = \langle \psi(\beta) | \hat{\mathcal{O}} | \psi(\beta) \rangle$ . This mean value can be considered as taken in a mixture involving density matrices with particle addition. Notice that this expectation can be rewritten in the form

$$\langle \hat{\mathcal{O}} \rangle_{\psi(\beta)} = \text{Tr}(\hat{\rho}_\psi \hat{\mathcal{O}}) \tag{10}$$

where

$$\hat{\rho}_\psi = \sum_{j,j' \in Z_2} \frac{a_j}{u(\beta)^j} \frac{a_{j'}^*}{u(\beta)^{j'}} \hat{c}^{\dagger j'} \hat{\rho}^{th} \hat{c}^j. \tag{11}$$

This density operator is also written as

$$\hat{\rho}_\psi = \frac{|a_0|^2}{u(\beta)^2} \hat{c}^\dagger \hat{\rho}^{th} \hat{c} + |a_1|^2 \hat{\rho}^{th} + \frac{a_0^* a_1}{u(\beta)} \hat{\rho}^{th} \hat{c} + \frac{a_0^* a_1}{u(\beta)} \hat{c}^\dagger \hat{\rho}^{th}. \tag{12}$$

The expectation value given in Eq. (10) represents a generalization of the thermofield expectation for the thermofield vacuum, corresponding to a superposition of thermofields associated directly to a density operator as given in Eq. (11).

The action of a non-thermalized gate operation on the state in Eq. (11) can be represented by an unitary operator  $\hat{\mathcal{U}}_G$  defining a new state

$$\hat{\rho}_\psi^{(G)} = \hat{\mathcal{U}}_G \hat{\rho}_\psi \hat{\mathcal{U}}_G^\dagger.$$

Then, explicitly  $\hat{\rho}_\psi^{(G)}$  reads

$$\hat{\rho}_\psi^{(G)} = \frac{|a_0|^2}{u(\beta)^2} \hat{c}_G^\dagger \hat{\rho}_G \hat{c}_G + |a_1|^2 \hat{\rho}_G + \frac{a_0^* a_1}{u(\beta)} \hat{\rho}_G \hat{c}_G + \frac{a_0^* a_1}{u(\beta)} \hat{c}_G^\dagger \hat{\rho}_G, \tag{13}$$

or, equivalently,

$$\hat{\rho}_\psi^{(G)} = \sum_{j,j' \in Z_2} \frac{a_j}{u(\beta)^j} \frac{a_{j'}^*}{u(\beta)^{j'}} \hat{c}_G^{\dagger j'} \hat{\rho}_G \hat{c}_G^j. \quad (14)$$

where now

$$\hat{\rho}_G = \hat{\mathcal{U}}_G \hat{\rho} \hat{\mathcal{U}}_G^\dagger, \quad (15)$$

$$\hat{c}_G = \hat{\mathcal{U}}_G \hat{c} \hat{\mathcal{U}}_G^\dagger. \quad (16)$$

This result implies that the action of a gate operation in the density operator associated in the thermofield qubit is obtained by the action of the same gate operation simultaneously in the density operator associated to the thermofield vacuum and the non-thermal creation and annihilation operators.

Taking the trace associated to the mean expectation of the operator  $\hat{\mathcal{O}}$ , we have, considering the unitarity of  $\hat{\mathcal{U}}_G$ ,

$$\begin{aligned} Tr(\hat{\rho}_\psi^{(G)} \hat{\mathcal{O}}) &= \frac{|a_0|^2}{u(\beta)} \langle \hat{\mathcal{U}}_G^\dagger \hat{\mathcal{O}} \hat{\mathcal{U}}_G c^\dagger \rangle \\ &+ |a_1|^2 \langle \hat{\mathcal{U}}_G^\dagger \hat{\mathcal{O}} \hat{\mathcal{U}}_G \rangle + \frac{a_0^* a_1}{u(\beta)} \langle \hat{\mathcal{U}}_G^\dagger \hat{\mathcal{O}} \hat{\mathcal{U}}_G \rangle \\ &+ \frac{a_0^* a_1}{u(\beta)} \langle \hat{\mathcal{U}}_G^\dagger \hat{\mathcal{O}} \hat{\mathcal{U}}_G \hat{c}^\dagger \rangle. \end{aligned}$$

Now let us consider the thermofield state resulting from the density operator under gate operation, by means of the identification

$$\langle \psi^{(G)}(\beta) | \hat{\mathcal{O}} | \psi^{(G)}(\beta) \rangle = Tr(\hat{\rho}_\psi^{(G)} \hat{\mathcal{O}}), \quad (17)$$

where  $|\psi^{(G)}(\beta)\rangle$  is the modified thermofield state resulting from the gate operation on the thermofield qubit. We can check the following

$$|\psi^{(G)}(\beta)\rangle = \sum_{j \in Z_2} a_j |j_G(\beta)\rangle,$$

where

$$|1_G(\beta)\rangle = \frac{\hat{c}_G^\dagger}{u(\beta)} |0_G(\beta)\rangle \quad (18)$$

and

$$\langle 0_G(\beta) | \hat{\mathcal{O}} | 0_G(\beta) \rangle = \text{Tr}(\rho_G \hat{\mathcal{O}}). \quad (19)$$

Finally, we can implement the correspondence with thermofield vacuum by means of

$$\text{Tr}(\rho_G \hat{\mathcal{O}}) = \text{Tr}(\rho \hat{\mathcal{U}}_G^\dagger \hat{\mathcal{O}} \hat{\mathcal{U}}_G) = \langle 0(\beta) | \hat{\mathcal{U}}_G^\dagger \hat{\mathcal{O}} \hat{\mathcal{U}}_G | 0(\beta) \rangle.$$

From this, we can finally make the identification

$$|0_G(\beta)\rangle = \hat{\mathcal{U}}_G |0(\beta)\rangle. \quad (20)$$

This result implies that the thermofield vacuum can be directly operated by the gate operation and that Eqs. (18) and (20) determine fully the action of the gate operator on the thermofield qubit, in complete agreement with the rearrangement of density operator and annihilation operators under the gate.

There are some subtleties that deserves to be analyzed. Under the gate operation the Bogoliubov relation is written as

$$\hat{c}_G^\dagger = u(\beta) \hat{\mathbf{c}}_G^\dagger(\beta) + v(\beta) \tilde{\mathbf{c}}_G(\beta), \quad (21)$$

with

$$\hat{\mathbf{c}}_G(\beta) = \hat{\mathcal{U}}_G \hat{\mathbf{c}}(\beta) \hat{\mathcal{U}}_G^\dagger, \quad (22)$$

$$\tilde{\mathbf{c}}_G(\beta) = \hat{\mathcal{U}}_G \tilde{\mathbf{c}}(\beta) \hat{\mathcal{U}}_G^\dagger. \quad (23)$$

The action into the thermofield vacuum now depends on the result of the action of the gate operation

$$\hat{c}_G^\dagger |0(\beta)\rangle = u(\beta) \hat{\mathbf{c}}_G^\dagger(\beta) |0(\beta)\rangle + v(\beta) \tilde{\mathbf{c}}_G(\beta) |0(\beta)\rangle. \quad (24)$$

We also have in the gate operated state, Eqs. (20) and (18),

$$\begin{aligned} \hat{c}_G^\dagger |0_G(\beta)\rangle &= u(\beta) \hat{\mathbf{c}}_G^\dagger(\beta) |0_G(\beta)\rangle + v(\beta) \tilde{\mathbf{c}}_G(\beta) |0_G(\beta)\rangle. \\ &= u(\beta) |1_G(\beta)\rangle. \end{aligned} \quad (25)$$

Then the state in Eq. (18) reads as

$$|1_G(\beta)\rangle = \hat{\mathbf{c}}_G^\dagger(\beta) |0_G(\beta)\rangle + \frac{v(\beta)}{u(\beta)} \tilde{\mathbf{c}}_G(\beta) |0_G(\beta)\rangle. \quad (26)$$



From Eqs. (24) and (25), the following commutation relations are fulfilled

$$[\hat{\mathcal{U}}_G, \hat{c}_G^\dagger] = u(\beta)[\hat{\mathcal{U}}_G, \hat{\mathbf{c}}^\dagger(\beta)] + v(\beta)[\hat{\mathcal{U}}_G, \tilde{\mathbf{c}}(\beta)].$$

Finally, from Eqs. (18) and (8), one has

$$\begin{aligned} |1_G(\beta)\rangle &= \frac{\hat{c}_G^\dagger}{u(\beta)} \hat{\mathcal{U}}_G |0(\beta)\rangle \\ &= \hat{\mathcal{U}}_G \frac{\hat{c}_G^\dagger}{u(\beta)} |0(\beta)\rangle \\ &= \hat{\mathcal{U}}_G |1(\beta)\rangle. \end{aligned} \tag{27}$$

This result shows that the action of the gate operator in the first excited thermofield state coincides with the first excited gate operated state.

### 3. Teleportation of thermofield qubits

Let us consider as a qubit state the following superposition of thermofield states, i. e., a thermofield qubit,

$$|\psi(\beta)\rangle_A = \sum_{j \in Z_2} a_j |j(\beta)\rangle_A, \tag{28}$$

where  $a_j$  are unknown complex numbers. The states  $|0(\beta)\rangle$  and  $|1(\beta)\rangle$  are thermofield vacuum and its first excitation, respectively<sup>35</sup>. The state given in Eq. (28) is prepared in Alice's lab.

A quantum channel composed of two entangled thermofield states, with a particle  $C$  belonging to Alice and a particle  $B$  belonging to Bob can be described by the following superposition of thermofield states

$$|\psi(\beta)\rangle_{BC} = \sum_{j \in Z_2} (-1)^j |j(\beta)\rangle_B |j+1(\beta)\rangle_C.$$

The whole state can be written as

$$|\psi(\beta)\rangle_{ABC} = \sum_{j, j' \in Z_2} (-1)^j a_{j'} |j'(\beta)\rangle_A |j(\beta)\rangle_B |j+1(\beta)\rangle_C.$$

The Alice's system composed of particles  $A$  and  $C$  can be conveniently rewritten in terms of a Bell basis of thermofield states, defined by

$$|\Psi_s(\beta)\rangle_{AC} = \sum_{j'' \in Z_2} s^{j''} |j''(\beta)\rangle_A |(j'' + 1)(\beta)\rangle_C, \quad (29)$$

$$|\Phi_s(\beta)\rangle_{AC} = \sum_{j'' \in Z_2} s^{j''} |j''(\beta)\rangle_A |j''(\beta)\rangle_C, \quad (30)$$

where  $s = \pm 1$ . With this in view, Alice projects the  $AC$  state in one of these Bell states, achieving at one of the following states

$${}_{AC}\langle\Psi_s(\beta)|\psi_{s'}(\beta)\rangle_{ABC} = \sum_{j \in Z_2} (-1)^j a_j s^j |j(\beta)\rangle_B, \quad (31)$$

$${}_{AC}\langle\Phi_s(\beta)|\psi_{s'}(\beta)\rangle_{ABC} = \sum_{j \in Z_2} (-1)^j a_{j+1} s^{j+1} |j(\beta)\rangle_B, \quad (32)$$

where the orthonormality relations lead to  $j'' = j' = j$  in Eq. (31) and  $j'' = j' = j + 1$  in Eq. (32).

Once Alice's decide in which basis realizing her measurement, she tells to Bob using a classical device what was her procedure. With such information Bob makes a corresponding projection choosing one of the following projectors

$$P_{s,s'}^\Psi = \sum_{j \in Z_2} (-1)^j s^j |j(\beta)\rangle\langle j(\beta)|, \quad (33)$$

$$P_{s,s'}^\Phi = \sum_{j \in Z_2} (-1)^j s^{j+1} |j+1(\beta)\rangle\langle j(\beta)|. \quad (34)$$

By implementing this procedure, Bob achieve at the following state

$$|\psi(\beta)\rangle_B = \sum_{j \in Z_2} a_j |j(\beta)\rangle_B, \quad (35)$$

corresponding to the teleportation of the thermofield state (28).

This procedure is a fidelity 1 quantum teleportation and makes use only of the algebraic properties of the thermofield states<sup>35</sup>. Due to this structure, Alice and Bob can also be in different temperatures or thermal baths,  $\beta$  and

$\beta'$ , such that the final state of Bob is a thermofield qubit of temperature  $\beta'^{-1}$  while Alice's has temperature  $\beta^{-1}$ . We will consider this point below.

Let us now consider the following initial states

$$\begin{aligned} |\psi\rangle_{00A} &= \sum_{j \in Z_2} a_j |j, \tilde{0}\rangle_A, \\ |\psi\rangle_{11A} &= \sum_{j \in Z_2} a_j |j, \tilde{1}\rangle_A, \\ |\psi\rangle_{01A} &= \sum_{j \in Z_2} a_j |j, \widetilde{j+1}\rangle_A, \\ |\psi\rangle_{10A} &= \sum_{j \in Z_2} a_j |j, \tilde{j}\rangle_A, \end{aligned}$$

defined in the space  $\mathcal{H} \otimes \tilde{\mathcal{H}}$ , but not in contact with a heat bath. Notice that although the states with subscripts 00, 11 are separable, the states 01 and 10 are entangled in  $\mathcal{H} \otimes \tilde{\mathcal{H}}$ . We can simplify in a matrix form

$$\begin{pmatrix} |\psi\rangle_{00A} & |\psi\rangle_{01A} \\ |\psi\rangle_{10A} & |\psi\rangle_{11A} \end{pmatrix}. \quad (36)$$

Alice has two particles that are not in a thermal bath, i.e., states at zero temperature. The Alice's particle  $C$  is shared in a quantum channel with Bob, that has a thermofield state, i.e., Bob's particle is in contact to a heat bath. Let us consider the quantum channel given by the following state

$$|\psi(\beta)\rangle_{BC} = \sum_{j' \in Z_2} (-1)^{j'} |j'(\beta)\rangle_B |j' + 1, \widetilde{j' + 1}\rangle_C.$$

The total state for each one of the initial states are given by

$$\begin{aligned} |\psi_{00}\rangle_{ABC} &= \sum_{j, j' \in Z_2} (-1)^{j'} a_j |j, \tilde{0}\rangle_A |j'(\beta)\rangle_B |j' + 1, \widetilde{j' + 1}\rangle_C, \\ |\psi_{11}\rangle_{ABC} &= \sum_{j, j' \in Z_2} (-1)^{j'} a_j |j, \tilde{1}\rangle_A |j'(\beta)\rangle_B |j' + 1, \widetilde{j' + 1}\rangle_C, \\ |\psi_{01}\rangle_{ABC} &= \sum_{j, j' \in Z_2} (-1)^{j'} a_j |j, \widetilde{j+1}\rangle_A |j'(\beta)\rangle_B |j' + 1, \widetilde{j' + 1}\rangle_C, \\ |\psi_{10}\rangle_{ABC} &= \sum_{j, j' \in Z_2} (-1)^{j'} a_j |j, \tilde{j}\rangle_A |j'(\beta)\rangle_B |j' + 1, \widetilde{j' + 1}\rangle_C. \end{aligned}$$

We have convenient 00-Bell basis for each one of the above  $AC$ -subsystems

$$|b_{s,00}^{(1)}\rangle_{AC} = \sum_{j'' \in Z_2} s^{j''} |j'', \tilde{0}\rangle_A |j'' + 1, \widetilde{j'' + 1}\rangle_C, \quad (37)$$

$$|b_{s,00}^{(2)}\rangle_{AC} = \sum_{j'' \in Z_2} s^{j''} |j'', \tilde{0}\rangle_A |j'', \widetilde{j''}\rangle_C, \quad (38)$$

where  $s = \pm$ .

Alice's projections in each one of this states will give

$$\langle b_{s,00}^{(1)} | \psi_{00} \rangle_{ABC} = \sum_{j \in Z_2} (-1)^j s^j a_j |j(\beta)\rangle_B, \quad (39)$$

$$\langle b_{s,00}^{(2)} | \psi_{00} \rangle_{ABC} = \sum_{j \in Z_2} (-1)^j s^{j+1} a_{j+1} |j(\beta)\rangle_B. \quad (40)$$

The 11-Bell basis is given by

$$|b_{s,11}^{(1)}\rangle_{AC} = \sum_{j'' \in Z_2} s^{j''} |j'', \tilde{1}\rangle_A |j'' + 1, \widetilde{j'' + 1}\rangle_C, \quad (41)$$

$$|b_{s,11}^{(2)}\rangle_{AC} = \sum_{j'' \in Z_2} s^{j''} |j'', \tilde{1}\rangle_A |j'', \widetilde{j''}\rangle_C. \quad (42)$$

Such that the projections lead to

$$\langle b_{s,11}^{(1)} | \psi_{11} \rangle_{ABC} = \sum_{j \in Z_2} (-1)^j s^j a_j |j(\beta)\rangle_B, \quad (43)$$

$$\langle b_{s,11}^{(2)} | \psi_{11} \rangle_{ABC} = \sum_{j \in Z_2} (-1)^j s^{j+1} a_{j+1} |j(\beta)\rangle_B. \quad (44)$$

The 01-Bell basis is given by

$$|b_{s,01}^{(1)}\rangle_{AC} = \sum_{j'' \in Z_2} s^{j''} |j'', j'' + 1\rangle_A |j'' + 1, \widetilde{j'' + 1}\rangle_C, \quad (45)$$

$$|b_{s,01}^{(2)}\rangle_{AC} = \sum_{j'' \in Z_2} s^{j''} |j'', j'' + 1\rangle_A |j'', \widetilde{j''}\rangle_C. \quad (46)$$

The projections lead to

$$\langle b_{s,01}^{(1)} | \psi_{01} \rangle_{ABC} = \sum_{j \in Z_2} (-1)^j s^j a_j |j(\beta)\rangle_B, \quad (47)$$

$$\langle b_{s,01}^{(2)} | \psi_{01} \rangle_{ABC} = \sum_{j \in Z_2} (-1)^j s^{j+1} a_{j+1} |j(\beta)\rangle_B. \quad (48)$$

Finally, the 10-Bell basis is given by

$$|b_{s,10}^{(1)}\rangle_{AC} = \sum_{j'' \in Z_2} s^{j''} |j'', \tilde{j}''\rangle_A |j'' + 1, \widetilde{j'' + 1}\rangle_C, \quad (49)$$

$$|b_{s,10}^{(2)}\rangle_{AC} = \sum_{j'' \in Z_2} s^{j''} |j'', \tilde{j}''\rangle_A |j'', \tilde{j}''\rangle_C, \quad (50)$$

with the projections leading to

$$\langle b_{s,10}^{(1)} | \psi_{10} \rangle_{ABC} = \sum_{j \in Z_2} (-1)^j s^j a_j |j(\beta)\rangle_B, \quad (51)$$

$$\langle b_{s,10}^{(2)} | \psi_{10} \rangle_{ABC} = \sum_{j \in Z_2} (-1)^j s^{j+1} a_{j+1} |j(\beta)\rangle_B. \quad (52)$$

When Alice tells to Bob in which basis she realized the projection, Bob can apply one of the projectors given in Eq. (33) or Eq. (34), achieving in the thermofield qubit

$$|\psi(\beta)\rangle_B = \sum_{j \in Z_2} a_j |j(\beta)\rangle_B.$$

The same idea can be used when Alice and Bob are in two different thermal baths with a shared quantum channel with sub-states at different temperatures. The presence of non-locality associated to the entanglement of quantum states at different temperatures is a still not well explored subject. Since the thermal interaction between two subsystems is a local effect, the non-local channel at different temperatures can be used for realizing quantum information protocols. In this point, the thermofield states give a clear difference between the non-local effect, given by the entanglement of the states at different temperatures, and the local effect that is the temperature associated to each subsystem.

#### 4. Changing the Mandel parameter of thermofield state under Gate operation

Let us consider the Mandel parameter in the case of thermofields, as discussed in a recent work<sup>82</sup>. Here we will consider how this parameter can

be changed under gate operation, in particular in the action in the thermofield vacuum and in a thermofield qubit. This quantity is described in terms of traces by means of the following

$$\mathcal{Q} = \frac{\text{Tr}(\hat{\rho}(\hat{n}^2 - \hat{n})) - [\text{Tr}(\hat{\rho}\hat{n})]^2}{\text{Tr}(\hat{\rho}\hat{n})},$$

where  $\hat{n} = \hat{c}^\dagger \hat{c}$ .

Under a gate operation, we have

$$\mathcal{Q}_G = \frac{\text{Tr}(\hat{\rho}_G(\hat{n}^2 - \hat{n})) - [\text{Tr}(\hat{\rho}_G\hat{n})]^2}{\text{Tr}(\hat{\rho}_G\hat{n})},$$

Using Eq. (17), then

$$\mathcal{Q}_G = \frac{\langle 0_G(\beta) | (\hat{n}^2 - \hat{n}) | 0_G(\beta) \rangle - \langle 0_G(\beta) | \hat{n} | 0_G(\beta) \rangle^2}{\langle 0_G(\beta) | \hat{n} | 0_G(\beta) \rangle}. \quad (53)$$

From Eq. (20) we can also write

$$\begin{aligned} \langle 0_G(\beta) | \hat{n} | 0_G(\beta) \rangle &= \langle 0(\beta) | \hat{\mathcal{U}}_G^\dagger \hat{n} \hat{\mathcal{U}}_G | 0(\beta) \rangle \\ &= \langle \hat{\mathcal{U}}_G^\dagger \hat{n} \hat{\mathcal{U}}_G \rangle. \end{aligned} \quad (54)$$

The action of the gate operation on the thermofield vacuum produces a change in the Mandel parameter given, finally, by

$$\mathcal{Q}_G = \frac{\langle \hat{\mathcal{U}}_G^\dagger (\hat{n}^2 - \hat{n}) \hat{\mathcal{U}}_G \rangle - \langle \hat{\mathcal{U}}_G^\dagger \hat{n} \hat{\mathcal{U}}_G \rangle^2}{\langle \hat{\mathcal{U}}_G^\dagger \hat{n} \hat{\mathcal{U}}_G \rangle}.$$

This result shows that the modified Mandel parameter depends on the gate operation  $\hat{\mathcal{U}}_G$ .

Now let us analyse the situation with the thermofield qubit. We can write the corresponding Mandel Parameter in the following way

$$\begin{aligned} \mathcal{Q}_G^\psi &= \frac{\langle \psi^{(G)}(\beta) | (\hat{n}^2 - \hat{n}) | \psi^{(G)}(\beta) \rangle}{\langle \psi^{(G)}(\beta) | \hat{n} | \psi^{(G)}(\beta) \rangle} \\ &\quad - \frac{\langle \psi^{(G)}(\beta) | \hat{n} | \psi^{(G)}(\beta) \rangle^2}{\langle \psi^{(G)}(\beta) | \hat{n} | \psi^{(G)}(\beta) \rangle}, \end{aligned} \quad (55)$$

which, by using Eq. (17), leads to

$$\mathcal{Q}_G^\psi = \frac{\text{Tr}(\hat{\rho}_\psi^{(G)}(\hat{n}^2 - \hat{n})) - \text{Tr}(\hat{\rho}_\psi^{(G)}\hat{n})^2}{\text{Tr}(\hat{\rho}_\psi^{(G)}\hat{n})},$$

Substituting the state given in Eq. (14), we write

$$\begin{aligned} \mathcal{Q}_G^\psi = & \left\{ \sum_{j,j' \in Z_2} \frac{a_j}{u(\beta)^j} \frac{a_{j'}^*}{u(\beta)^{j'}} \langle \hat{\mathcal{U}}_G^\dagger \hat{c}_G^j (\hat{n}^2 - \hat{n}) \hat{c}_G^{\dagger j'} \hat{\mathcal{U}}_G \rangle \right. \\ & \left. - \left[ \sum_{j,j' \in Z_2} \frac{a_j}{u(\beta)^j} \frac{a_{j'}^*}{u(\beta)^{j'}} \langle \hat{\mathcal{U}}_G^\dagger \hat{c}_G^j \hat{n} \hat{c}_G^{\dagger j'} \hat{\mathcal{U}}_G \rangle \right]^2 \right\} \\ & \times \frac{1}{\sum_{j,j' \in Z_2} \frac{a_j}{u(\beta)^j} \frac{a_{j'}^*}{u(\beta)^{j'}} \langle \hat{\mathcal{U}}_G^\dagger \hat{c}_G^j \hat{n} \hat{c}_G^{\dagger j'} \hat{\mathcal{U}}_G \rangle}, \end{aligned}$$

This result shows that the action of the gate operation in a thermofield qubit produces a change in the Mandel parameter corresponding to the action of the unitary operator  $\hat{\mathcal{U}}_G$  in the density operator associated to the thermofield qubit, producing changes in the expectation values.

## 5. Gibbs-like density operator and gate operator effect

As it has been shown earlier<sup>87</sup>, a Hamiltonian in the simple form  $\hat{H} = \omega \hat{n}$  has in its thermofield form a thermofield vacuum associated with a corresponding Gibbs-like density operator  $\hat{\rho} = e^{-\beta \omega \hat{n}} / \mathcal{Z}$ , with  $\mathcal{Z} = \text{Tr}(e^{-\beta \hat{H}})$ . Using the results above we have that the modified state under the action of a gate operation is given by

$$\hat{\rho}_G = \frac{1}{\mathcal{Z}} \hat{\mathcal{U}}_G \exp(-\beta \omega \hat{n}) \hat{\mathcal{U}}_G^\dagger.$$

The corresponding thermofield qubit is now written in following way

$$\hat{\rho}_\psi^{(G)} = \sum_{j,j' \in Z_2} \frac{a_j}{u(\beta)^j} \frac{a_{j'}^*}{u(\beta)^{j'}} \hat{c}_G^{\dagger j'} \frac{1}{\mathcal{Z}} \hat{\mathcal{U}}_G \exp(-\beta \omega \hat{n}) \hat{\mathcal{U}}_G^\dagger \hat{c}_G^j. \quad (56)$$

Consider this problem in the context of a spin 1/2 system. In terms of the basis of spin, we can write<sup>35</sup>

$$\hat{\rho} = \frac{1}{\mathcal{Z}} e^{-\beta\omega\hat{S}_0} \left| \frac{1}{2} \right\rangle \left\langle \frac{1}{2} \right| + \frac{1}{\mathcal{Z}} e^{-\beta\omega\hat{S}_0} \left| -\frac{1}{2} \right\rangle \left\langle -\frac{1}{2} \right|,$$

such that  $\hat{S}_0|\sigma\rangle = \sigma|\sigma\rangle$ ,  $\sigma = \pm 1/2$ . Taking a gate operation as a Hadamard operation over the states, we have  $\hat{\mathcal{U}}_{Hadamard}|\pm\frac{1}{2}\rangle = \frac{1}{\sqrt{2}}(|\frac{1}{2}\rangle \pm |-\frac{1}{2}\rangle)$  leading to the following modified state under this gate operation

$$\hat{\rho}^{(Hadamard)} = \frac{1}{2\mathcal{Z}} \sum_{s=\pm} \exp\left(\frac{s\beta\omega}{2}\right) \times \left(|\frac{1}{2}\rangle + s|-\frac{1}{2}\rangle\right) \left(\langle\frac{1}{2}| + s\langle-\frac{1}{2}|\right).$$

It is also important to notice that this operation is reversible, such that we can restore the original state by applying a second Hadamard gate

$$\hat{\rho} = \hat{\mathcal{U}}_{Hadamard} \hat{\rho}^{(Hadamard)} \hat{\mathcal{U}}_{Hadamard}^\dagger.$$

This implies that one can also start with an operated gate state and recover the corresponding thermofield state by applying an adequate reversible gate.

## 6. No-cloning for thermofields

Consider the action of the tilde conjugation defined by the action of a doubling map

$$\mathcal{D}_{TFD}(|0\rangle) = |0, \tilde{0}\rangle. \quad (57)$$

There is an association between the doubling procedure in TFD and the no-cloning theorem<sup>81</sup>. The doubling procedure has the same characteristics involved in the no-cloning theorem: it cannot be realized unitarily for an arbitrary superposition state, since the requirement of linearity cannot be achieved. As such, the extension

$$\mathcal{D}_{TFD}(|\psi\rangle) = |\psi, \tilde{\psi}\rangle, \quad (58)$$

where  $|\psi\rangle$  is a qubit state, is not implemented via unitary operation. This does not constitute a problem for TFD itself since it starts from the vacuum and generates the doubling vacuum for applying a temperature dependent Bogoliubov operation. However, an extension of the method starting from



the doubling of arbitrary states is forbidden under linearity requirement. Consequently, Eq. (58) is not valid in general, but Eq. (57) is fully consistent.

Another important point here is the no-cloning theorem for thermofield qubits. Consider a cloning map

$$\mathcal{C}(\ast) = \ast \otimes \ast, \quad (59)$$

where  $\ast$  is an arbitrary algebraic quantity. Let us consider the thermofield qubit *inside* this map. We verify that

$$\begin{aligned} \mathcal{C}\left(\sum_{j \in Z_2} a_j |j(\beta)\rangle\right) &= \left(\sum_{j \in Z_2} a_j |j(\beta)\rangle\right) \otimes \left(\sum_{j' \in Z_2} a_{j'} |j'(\beta)\rangle\right). \\ &= \sum_{j, j' \in Z_2} a_j a_{j'} |j(\beta)\rangle \otimes |j'(\beta)\rangle. \end{aligned} \quad (60)$$

But this map is not linear since the linearity requires

$$\sum_{j \in Z_2} a_j \mathcal{C}(|j(\beta)\rangle) = \sum_{j \in Z_2} a_j |j(\beta)\rangle \otimes |j(\beta)\rangle,$$

that is not equivalent to Eq. (60):

$$\mathcal{C}\left(\sum_{j \in Z_2} a_j |j(\beta)\rangle\right) \neq \sum_{j \in Z_2} a_j \mathcal{C}(|j(\beta)\rangle).$$

This result implies that a thermofield qubit cannot be cloned, in complete agreement with the no-cloning theorem, at zero temperature.

## 7. Maps for connecting thermofield vacua, no-broadcasting theorem and superposition of thermofield vacua

Consider the following map

$$\mathcal{T}(\hat{\rho}) = \hat{\rho}', \quad (61)$$

such that

$$\langle 0(\beta) | \hat{\mathcal{O}} | 0(\beta) \rangle = \text{Tr}(\hat{\rho} \hat{\mathcal{O}}), \quad (62)$$

and

$$\langle 0(\beta') | \hat{\mathcal{O}} | 0(\beta') \rangle = \text{Tr}(\hat{\rho}' \hat{\mathcal{O}}). \quad (63)$$

This map take a thermofield vacuum associated to a given temperature  $\beta^{-1}$  and leads to another thermofield vacuum associated to other temperature  $\beta'^{-1}$ .

In particular, the zero temperature state leads to

$$\langle 0(\infty) | \hat{\mathcal{O}} | 0(\infty) \rangle = Tr(|0\rangle\langle 0| \hat{\mathcal{O}}). \quad (64)$$

Physically, this process has to be associated to an exchange of thermal baths.

A whole density matrix incorporating all the associated temperatures can be described by

$$\hat{\rho}^{(\mathcal{T})} = \int d\beta \mu_\beta \hat{\rho}_\beta.$$

such that

$$Tr(\hat{\rho}^{(\mathcal{T})} \hat{\mathcal{O}}) = \int d\beta \mu_\beta \langle 0(\beta) | \hat{\mathcal{O}} | 0(\beta) \rangle.$$

This state discribes the passage for all temperatures, and the association to a given temperature  $\beta_0$  comes from the association  $\mu_\beta = \delta(\beta - \beta_0)$  and

$$\hat{\rho}_{\beta_0} = \int d\beta \delta(\beta - \beta_0) \hat{\rho}_\beta.$$

We now return to the map in Eq. (61). Applying in the above state it leads to

$$\mathcal{T}(\hat{\rho}_{\beta_0}) = \int d\beta \tilde{\mathcal{T}}(\delta(\beta - \beta_0)) \hat{\rho}_\beta = \hat{\rho}_{\beta'},$$

and consequently,

$$\tilde{\mathcal{T}}(\delta(\beta - \beta_0)) = \delta(\beta - \beta').$$

Now, consider a doubling procedure of a Hilbert space

$$\mathcal{H} \rightarrow \mathcal{H}_A \otimes \mathcal{H}_B$$

and a map for density matrices  $\hat{\rho}$

$$\mathcal{B}(\hat{\rho}) \in \mathcal{H}_A \otimes \mathcal{H}_B,$$

with  $\hat{\rho} \in \mathcal{H}$ , i.e. a given state for the original Hilbert space is mapped in a density-like matrix in the doubled space. If

$$Tr_A(\mathcal{B}(\hat{\rho})) = Tr_B(\mathcal{B}(\hat{\rho})) = \hat{\rho},$$

we say that the  $\mathcal{B}(\hat{\rho})$  broadcasts  $\hat{\rho}$ .

The cloning map, Eq. (59), broadcasts  $\hat{\rho}$ , but the lack of linearity implies that superpositions like  $\hat{\rho} = \sum_{j \in Z_2} a_j \rho_j$  cannot be cloned and, as such are not broadcasted by the cloning map.

As in the usual no-cloning case, there is a no-broadcasting theorem that asserts that the above procedure cannot be achieved for an arbitrary density matrix<sup>88</sup>. Indeed, it is not difficult to find examples of where broadcasting is not achieved. Consider

$$\rho' = \mu \rho \otimes |0\rangle\langle 0| + (1 - \mu)|0\rangle\langle 0| \otimes \rho, \quad (65)$$

where  $\mu \in [0, 1]$ , and  $\rho$  is the density operator associated to the thermofield vacuum, the partial traces are given by  $Tr_A(\rho') = \mu|0\rangle\langle 0| + (1 - \mu)\rho$  and  $Tr_B(\rho') = \mu\rho + (1 - \mu)|0\rangle\langle 0|$  and consequently  $\rho'$  do not comes from a broadcasting. On the other hand, other possible state

$$\rho'' = \mu \rho \otimes \rho + (1 - \mu)|0\rangle\langle 0| \otimes |0\rangle\langle 0|,$$

has  $Tr_A(\rho'') = Tr_B(\rho'') = \mu\rho + (1 - \mu)|0\rangle\langle 0|$ , and consequently is a candidate to broadcasted state.

We can introduce a more general notion of broadcasting in the thermofield context, where although we do not require the complete broadcasting, we require that a map leads to two different density operators associated with thermofield vacua at different temperatures. We can write

$$Tr_A(\mathcal{B}(\hat{\rho}_\beta)) = \mathcal{T}(\hat{\rho}_\beta) = \hat{\rho}_{\beta'} \quad (66)$$

$$Tr_B(\mathcal{B}(\hat{\rho}_\beta)) = \mathcal{T}'(\hat{\rho}_\beta) = \hat{\rho}_{\beta''} \quad (67)$$

where  $\mathcal{T}$  and  $\mathcal{T}'$  are maps in Eq. (61), that connect the new density operators associated to thermofield vacua at temperatures  $\beta'^{-1}$  and  $\beta''^{-1}$ , respectively. Then the state in Eq. (65) is associated with such a transformation, where the cases  $\mu = 1$  and  $\mu = 0$  represent the extremal,  $Tr_A(\rho') = \rho$  and  $Tr_B(\rho') = |0\rangle\langle 0|$ , respectively.

Let us consider the state  $\mu\rho + (1 - \mu)|0\rangle\langle 0|$  as the passage of a finite temperature state  $\hat{\rho}$  to a zero temperature state  $|0\rangle\langle 0|$ , such that the state corresponds to a mixture state of two different thermal baths. Let us consider a more general case  $\mu\rho_\beta + (1 - \mu)\rho_{\beta'}$ . Taking the trace, we have

$$\mu Tr(\hat{\rho}_\beta \hat{\mathcal{O}}) + (1 - \mu) Tr(\hat{\rho}_{\beta'} \hat{\mathcal{O}}),$$

that associates two mean expectation values with the observable  $\hat{\mathcal{O}}$  for different temperatures, such that

$$\mu\langle\hat{\mathcal{O}}\rangle_{\beta} + (1 - \mu)\langle\hat{\mathcal{O}}\rangle_{\beta'}.$$

This description is of particular interest in metastable states or non-equilibrium situation, where the states are connected by means of two temperatures near to each other. The corresponding thermofield state associated to this case is

$$|0(\beta, \beta')\rangle = \sqrt{\mu}|0(\beta)\rangle + \sqrt{(1 - \mu)}|0(\beta')\rangle.$$

States at different temperatures are orthonormal, and are written as

$$\langle 0(\beta, \beta')|\hat{\mathcal{O}}|0(\beta, \beta')\rangle = \mu\langle\hat{\mathcal{O}}\rangle_{\beta} + (1 - \mu)\langle\hat{\mathcal{O}}\rangle_{\beta'}.$$

This relation gives us another prospect for the description of states with different vacua at finite temperature involved. Although in a non-equilibrium case a given temperature is not an appropriate parameter, we can consider superpositions of thermofield vacua at different temperatures, in such a way that the expectation value of an observable with respect to  $|0(\beta, \beta')\rangle$  provides a measurable estimation.

## 8. Conclusions

In this paper, we have considered quantum information protocols involving thermofield superpositions of the thermal vacuum and its first excitation in the non-tilde sector, a thermofield qubit. We derived a generalized expectation for thermofield qubits and considered the action of gate operations. We also proposed QT protocols involving thermofield states. The QT of thermofield qubits was implemented exploring the algebraic properties of these states, by means of which we have incorporated the presence of temperature naturally, according to TFD procedure<sup>36</sup>. With this approach, we also discussed the case where Alice and Bob are in different temperatures and share a non-local channel of entangled thermofield states with two different temperatures. Our results show that quantum teleportation can be achieved even if Alice and Bob are in different temperatures. As a particular case for the action of gate operations, we considered Gibbs-like density operators under gate actions. By considering the Mandel parameter for thermofield states, we also discussed its changing under gate operations. We also have

discussed no-cloning theorem in TFD<sup>81</sup> and considered the non-broadcasting problem for a thermal context, where we considered maps connecting thermofield vacua, no-broadcasting theorem<sup>88</sup> and superposition of thermofield vacua and thermofield states at different temperatures. Metastable and non-equilibrium scenarios were also discussed.

One important aspect to be explored in a more fundamental point is the relation of temperature and non-locality. By exploring this point with TFD approach we can circumvent some difficulties associated to the inclusion of temperature by indirect means. In fact, we considered superpositions of thermofield states in protocols where thermal baths at different temperatures are linked by non-local channels. The locality of thermal effects, in the sense that they are localized to a given region and cannot be moved by non-local effects is an important point that has to be explored in more detail in another place. Some neglected points as dissipative effects affecting dynamically thermofield qubits states will be also explored in somewhere else, although some previous proposals have touched this question<sup>24,89,90</sup>.

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